# The proliferation of centenarians in Brazil: Indirect estimations using alternative approaches♠

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## Introduction

The proliferation of centenarians is a significant feature of the aging process. In today's low-mortality countries, the number of centenarians more than doubled each decade since 1950 (Jeune and Kannisto 1997). On average, the number of new centenarians increased at an annual rate of about 7% between 1950s and 1980s in these countries (Vaupel and Jeune 1995). In countries with higher mortality, like Brazil, the multiplication of centenarians has been observed as well. In Brazil, according to census data, there were 13,296 centenarians in 1991 and 24.236 in 2010 (IBGE 1991, 2010). However, due to data quality issues, these numbers are questionable. Accurate estimates of the number of the oldest-old population are important for correctly assessing the current scale of the aging process and predicting future demographic and socioeconomic changes.

In any population, the growth rate of centenarian population must be due to some combination of earlier changes in fertility, migration and mortality (Preston and Coale 1982). According to Vaupel and Jeune (1995), improvements in survival from age 80 to 100 is the main cause of the proliferation of centenarians, explaining about 70% of the growth rate of centenarian population in Northern and Western-central Europe from the 1970s to 1980s. The case of Japan is a good example of this aspect. Japanese population experienced high rates of mortality improvement for ages 80-99 from 1960s to the 1980s (Kannisto et al. 1994), which probably triggered the increased number of centenarians from 153 people in 1963 to 40,399 people in 2009 (Saito 2005, Robine et al. 2010). In Brazil, average death rates of octogenarians have declined at a rate of 2.9% per year for males and 3.5% per year for females in the 1980s (Campos 2004). Indirect estimates of mortality rates also suggested mortality reduction at ages above 80 in the period 1980 to 2000 (Agostinho 2009). Mostly of this mortality decline is a result of economic developments, investments in public health and advances in medical technology. Moreover, improvements in survival at older ages over the past decades certainly contributed to the proliferation of centenarians in Brazil.

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A simple comparison between the Brazilian proportions of centenarians over the years makes us skeptical about the population counts reported by the census bureau, suggesting variations in data quality across censuses. According to census data, the centenarian share of the total population was 0.91 per 10,000 in 1991, 1.45 per 10,000 in 2000 and 1.27 per 10,000 in 2010 (IBGE 1991, 2000, 2010). Due to declining mortality at older ages over decades, we expected an increasing proportion of centenarians in Brazil. Probably, the decline in proportion of centenarians between 2000 and 2010 is a result of progress in census data quality, such as improvements in age reported caused by higher education at advanced ages.

Another simple way to check the quality of the Brazilian centenarian data is comparing the prevalence of centenarians in Brazil with a country with presumably higher data quality and lower mortality levels. In Sweden, for instance, the centenarian share of the total population in 1991 (0.69 per 10.000) was about 2/3 the Brazilian proportion in 1991 (HMD 2016). Conversely, the Swedish proportion of centenarians in 2010 (1.72 per 10.000) is 35% higher than that of Brazil in 2010 (HMD 2016). This improbably higher proportion of older population in the 1991 census in Brazil could be a result of the poor quality data. However, the comparison in 2010 suggests census data improvements in Brazil.

Official population estimates are especially problematic at advanced ages. Therefore, studies of the centenarian population must take into account data quality issues. In many populations, the most important problem of census data at older ages is age misstatement (Preston et al. 1999). In Brazil, the age reporting in the 1991, 2000 and 2010 census is derived from two questions. The first one asked the month and year of birth of the person. The second question asked for the presumably age in years and months and is used when the person do not know her date of birth. In the absent of these two answers, information of age is imputed by the census bureau. Using the presumably age, the error of digit preference can emerge. Brazilians tend to prefer the ending age digits zero and five (Horta 2012; Horta, Sawyer e Carvalho 2006). Both problems of digit preference and of missing data are very significant among the oldest-old population (Popolo 2001, Gomes and Turra 2009). The problem of age exaggeration is also common at advanced ages (Popolo 2001, Dechter and Preston 1991). All the problems mentioned above are believed to be responsible for an overstatement of the number of centenarians in Brazil. Moreover, the relatively small number of centenarians in a population can cause data to be particularly sensitive to these data issues.

The poor data quality at advanced ages increases the uncertainty of the true number of centenarians recorded in the Brazilian census bureau. Therefore, indirect methods are very helpful to evaluate the quality of census data, and to estimate the number of centenarians closer to the true size of the oldest-old population. In Brazil, indirect estimates based on the extinct generation method revealed that the centenarians recorded in the 1991 census are exaggerated (Gomes and Turra 2009). This study suggested that the number of centenarians estimated indirectly is one third the number recorded by the 1991 census in Brazil. Nevertheless, few studies have analyzed centenarians in Brazil. Indeed, little is known about the true proliferation of centenarians in Brazil during the last decades.

In order to complement analysis of centenarians in Brazil, we estimate the population aged 100 to 109 years by sex in 1991, 2000 and 2010 using three estimation procedures. First we use a method developed by Wilmoth (1995) to compute the proportion of centenarians in a stable population. The second estimation procedure is a variant of Wilmoths' method. Instead of assume a stable population, we compute the proportion of centenarians in a non-stable population, considering a set of age-specific growth rates. Finally, we estimate the centenarian population "projecting" the population at one census date to a later date using the forward-survival method (Shryock and Siegel 1976).

## Methods and Data

In this study we estimate the number of people at age group 100-109 by sex, for 1991, 2000 and 2010. To do that, three estimation procedures are used: (*i*) the Wilmoth's method assuming a stable population; (*ii*) the Wilmoth's method assuming a non-stable population; and (*iii*) the cohort reconstruction through the forward-survival method.

### *i. First estimation procedure*

First, we use the indirect method develop by Wilmoth (1995). This method computes the proportion of centenarian using a mortality model to estimate the survival probability from a given adult age $y$ to 100.

The first step of the Wilmoths' method is to choose an age pattern of mortality to estimate the survival probability from a given adult age $y$ to age 100. There is a general agreement that mortality increases exponentially, as described by the Gompertz curve, for mid-adult and early old ages. However, there is not a consensus regarding the mortality trajectory at the most advanced ages (Gavrilov and Gavrilova 2015; Gavrilov and Gavrilova 2011; Gampe 2010; Robine et al. 2005; Robine and Vaupel 2001). Some studies suggested that the exponential growth of mortality with age is followed by a period of deceleration, with slower rates of mortality increase at the oldest ages (Gampe 2010; Robine and Vaupel 2001; Robine et al. 2005). Conversely, another group of researchers pointed out that the mortality deceleration in later life is more expressed for data with lower quality, and hence mortality continues to grow exponentially at the highest ages (Gavirilov and Gavirilova 2011, 2015). Due to this disagreement about the mortality trajectory at the most advanced ages, we decide to assume that mortality slows down at the most advanced ages, as assumed by Wilmoth (1995). Moreover, we assume that the age 50 (*y*=50) is the starting point for our model of adult mortality. This choice can be justified by the importance of adult mortality in centenarians' mortality (Vaupel and Jeune 1995).

The Gompertz-Perks curve follows a logistic form, and it is a modification of the well-known Gompertz mortality curve. The Gompertz-Perks curve deviates from the Gompertz model at both younger and older adult ages, and it contains an inflection point in late adulthood that should move upward as mortality falls. The Gompertz-Perks formula contains 4 parameters:

$$\mu(x) = \frac{c + ae^{bx}}{1 + vae^{bx}} \qquad (1)$$

where $c \geq 0$ represents the level of background mortality, and $1/v$ gives the upper asymptote of the mortality curve. The parameters *a* and *b* are more abstract, where $b > 0$ is the rate of exponential increase in mortality across the age range in the Gompertz model, but this interpretation is only approximate in the Gompertz-Perks model (Wilmoth 1995), and $a > 0$ is the exact force of mortality at age 0 only in the Gompertz model, but this fact does not aid in interpretation since the model applies to adult mortality alone (Wilmoth 1995).

To lend these parameters more direct interpretations, Wilmoth (1995) proposed a re-parametrization of the Gompertz-Perks model. According to the author, the remaining life expectancy at the starting age for the late adult mortality model, $e_{50}$, is an alternative for the parameter *a*, and the death rate between ages 50 and 55 (${}_5m_{50}$) is chosen as alternative for the parameter *b* in the formula (1). Using numerical methods[3] we find the unique parameters, *a* and *b*, in the formula (1) that reproduce a given $e_{50}$ and ${}_5m_{50}$. Thus, all model assumptions in this study are expressed in terms of $e_{50}$, ${}_5m_{50}$, *c* and *v*.

a) Choosing the mortality levels for 1991, 2000 and 2010

Having chosen the mortality curve, it is necessary to select the parameters of the Gompertz-Perks model. We start selecting the mortality levels: (1) life expectancy at birth ($e_0$), and (2)

[3] We used numerical optimization to find the maximum likelihood estimate of the parameter *a* given *b* using the equation (1) and assuming that $\mu(52.5) = {}_5m_{50}$.

the remaining life expectancy at age 50 ( $e_{50}$ ). To select these values we use a combination of sources in order to calculate the average of $e_0$ and $e_{50}$. We believe to achieve more accurate values of $e_0$ and $e_{50}$ through a combination of sources.

We use the life expectancy at birth as a reference measure to select life tables of countries with better data quality than Brazil. To define the $e_0$ for 1991, 2000 and 2010 in Brazil, we use Brazilian life tables estimated by: (a) the Brazilian Institute of Geography and Statistics (IBGE) for 1991, 2000 and 2010, (b) Agostinho (2009) for 1991 and 2000, (c) the World Health Organization for 1990 and 2000, (d) and by the Centre for Development and Regional Planning (CEDEPLAR) for 2010. The average life expectancy at birth based on these sources is: 70.4 years for female and 63.6 years for male in 1991, 74.0 years for female and 67.0 years for male in 2000, and 77.2 years for female and 70.3 for male in 2010. Thus, we consider these values as more accurate estimates of life expectancy at birth in Brazil.

After define $e_0$, we choose $e_{50}$ (parameter *a*) for 1991, 2000 and 2010 in Brazil. Instead of use only Brazilian life tables we also use life tables from several countries with presumably higher quality data[4] than Brazil to calculate the average of $e_{50}$. The life expectancy at birth defined to Brazil, are our reference measure to select life tables of low-mortality countries. For instance, the female $e_0$ in 2010 in Brazil (77.2 years) it is close to the female $e_0$ in 1970 in Sweden (77.2 years). Thus, the Swedish life table for females in 1970 is used to determine the $e_{50}$ in 2010 for females in Brazil. Thus, the average of $e_{50}$ from the selected life tables is believed to be a more accurate estimate than the $e_{50}$ based on only Brazilian life tables. Our strategy here is to reduce the effects of data quality issues on mortality rates at older ages in Brazil[5].

Tables 1 and 2 present the values of $e_0$ and $e_{50}$ used to define the remaining life expectancy at age 50 in 1991, 2000 and 2010 in Brazil, respectively for females and males.

From these results, we assume that the remaining life expectancy at age 50 in 1991, 2000, and 2010 in Brazil are the average of $e_{50}$ derived from the selected life tables (Tables 1 and 2). Thus, the $e_{50}$ is around 26 years for females and 22 years for males in 1991, around 27 years

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[4] We selected life tables of today's low-mortality countries with $e_0$ in the range: $mean(e_0^{Brazil}) - standard\ deviation(e_0^{Brazil}) \leq e_0^{low-mortality\ countries} \leq mean(e_0^{Brazil}) + standard\ deviation(e_0^{Brazil})$

[5] Number of deaths is not recorded quite precisely in Brazil. Moreover, age misreporting also affects census data, especially at older ages.

for females and 23 years for males in 2000, and around 30 years for females and 24 for males in 2010.

**Table 1**: Life expectancies at birth ($e_0$) and at age 50 ($e_{50}$) and proportion of surviving at age 50 ($l(50)$) in the selected low-mortality countries and Brazil, female.

| Country | Reference year to estimate the centenarians in Brazil | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1991 | | | | 2000 | | | | 2010 | | | |
| | Year | $e_0$ | $e_{50}$ | $l(50)$ | Year | $e_0$ | $e_{50}$ | $l(50)$ | Yearr | $e_0$ | $e_{50}$ | $l(50)$ |
| Brazil | 1991 | 70.5 | 26.3 | 0.8917 | 2000 | 74.0 | 27.7 | 0.9298 | 2010 | 77.3 | 29.9 | 0.9513 |
| Sweden | 1945 | 69.5 | 26.1 | 0.8717 | 1954 | 73.9 | 27.2 | 0.9330 | 1970 | 77.2 | 29.8 | 0.9496 |
| Sweden | 1946 | 70.7 | 26.2 | 0.8906 | 1955 | 74.2 | 27.6 | 0.9333 | 1971 | 77.4 | 29.9 | 0.9501 |
| Sweden | 1947 | 70.6 | 25.9 | 0.8932 | - | - | - | - | 1972 | 77.5 | 30.0 | 0.9523 |
| Italy | 1954 | 69.9 | 27.0 | 0.8799 | 1968 | 73.7 | 28.1 | 0.9225 | 1978 | 77.1 | 29.9 | 0.9501 |
| Italy | 1955 | 70.4 | 27.1 | 0.8853 | 1969 | 73.9 | 28.2 | 0.9225 | 1979 | 77.5 | 30.1 | 0.9516 |
| Italy | 1956 | 69.8 | 26.2 | 0.8886 | 1970 | 74.9 | 28.9 | 0.9225 | 1981 | 77.8 | 30.2 | 0.9542 |
| Italy | 1957 | 70.1 | 26.8 | 0.8861 | 1971 | 73.7 | 27.4 | 0.9225 | - | - | - | - |
| Italy | 1958 | 71.2 | 27.5 | 0.8937 | - | - | - | - | - | - | - | - |
| Japan | 1959 | 69.8 | 26.2 | 0.8788 | 1966 | 73.7 | 27.4 | 0.9258 | 1977 | 77.9 | 30.2 | 0.95487 |
| Japan | 1960 | 70.2 | 26.0 | 0.8873 | 1967 | 74.0 | 27.6 | 0.9281 | - | - | - | - |
| Japan | 1961 | 70.8 | 26.2 | 0.8960 | 1968 | 74.3 | 27.7 | 0.9320 | - | - | - | - |
| Japan | 1962 | 71.1 | 26.2 | 0.9021 | 1969 | 74.6 | 27.9 | 0.9335 | - | - | - | - |
| Switzerland | 1948 | 69.4 | 25.5 | 0.8827 | 1958 | 73.9 | 27.6 | 0.9306 | 1974 | 77.6 | 30.1 | 0.95229 |
| Switzerland | 1949 | 70.0 | 25.6 | 0.8933 | - | - | - | - | - | - | - | - |
| Switzerland | 1950 | 71.1 | 26.2 | 0.9015 | - | - | - | - | - | - | - | - |
| Switzerland | 1951 | 71.0 | 26.0 | 0.9020 | - | - | - | - | - | - | - | - |
| Portugal | 1968 | 69.9 | 27.4 | 0.8781 | 1977 | 73.9 | 28.4 | 0.9195 | 1986 | 77.1 | 30.2 | 0.9431 |
| Portugal | 1970 | 70.1 | 27.5 | 0.8804 | - | - | - | - | - | - | - | - |
| UK | 1949 | 70.3 | 26.1 | 0.8879 | 1963 | 73.7 | 27.3 | 0.9290 | 1983 | 77.2 | 29.5 | 0.95472 |
| UK | 1950 | 70.9 | 26.2 | 0.8972 | - | - | - | - | 1984 | 77.6 | 29.8 | 0.95604 |
| UK | 1951 | 70.7 | 25.7 | 0.9011 | - | - | - | - | | | | |
| Finland | 1955 | 70.7 | 25.5 | 0.9041 | 1970 | 74.4 | 27.2 | 0.9426 | - | - | - | - |
| Finland | 1956 | 71.2 | 25.8 | 0.9072 | - | - | - | - | - | - | - | - |
| Finland | 1957 | 70.6 | 25.5 | 0.9036 | - | - | - | - | - | - | - | - |
| Denmark | 1947 | 69.6 | 25.7 | 0.8822 | 1956 | 73.7 | 27.5 | 0.9271 | - | - | - | - |
| Denmark | - | - | - | - | 1957 | 73.5 | 27.3 | 0.9261 | - | - | - | - |
| Denmark | - | - | - | - | 1958 | 74.0 | 27.5 | 0.9321 | - | - | - | - |
| Denmark | - | - | - | - | 1959 | 73.9 | 27.6 | 0.9287 | - | - | - | - |
| Denmark | - | - | - | - | 1960 | 74.0 | 27.5 | 0.9316 | - | - | - | - |
| Denmark | - | - | - | - | 1961 | 74.4 | 28.0 | 0.9312 | - | - | - | - |
| Denmark | - | - | - | - | 1962 | 74.4 | 27.8 | 0.9344 | - | - | - | - |
| Belgium | 1952 | 70.5 | 26.4 | 0.8932 | 1965 | 73.6 | 27.5 | 0.9265 | 1982 | 77.2 | 29.7 | 0.94933 |
| Belgium | 1953 | 70.9 | 26.4 | 0.9000 | 1966 | 73.7 | 27.5 | 0.9263 | 1983 | 77.2 | 29.7 | 0.94857 |
| Belgium | 1954 | 71.1 | 26.6 | 0.9006 | 1967 | 74.1 | 27.7 | 0.9304 | 1984 | 77.8 | 30.2 | 0.95174 |
| Belgium | - | - | - | - | 1968 | 73.8 | 27.3 | 0.9324 | - | - | - | - |
| Belgium | - | - | - | - | 1969 | 74.0 | 27.5 | 0.9312 | - | - | - | - |
| Belgium | - | - | - | - | 1970 | 74.2 | 27.7 | 0.9315 | - | - | - | - |
| Belgium | - | - | - | - | 1971 | 74.4 | 27.8 | 0.9340 | - | - | - | - |

Source: HMD data.

**Table 2**: Life expectancies at birth ($e_0$) and at age 50 ($e_{50}$) and proportion of surviving at age 50 ($l(50)$) in the selected low-mortality countries and Brazil, male.

| Country | Reference year to estimate the centenarians in Brazil | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **1991** | | | | **2000** | | | | **2010** | | | |
| | **Year** | $e_0$ | $e_{50}$ | $l(50)$ | **Year** | $e_0$ | $e_{50}$ | $l(50)$ | **Yearr** | $e_0$ | $e_{50}$ | $l(50)$ |
| Brazil | 1991 | 63.6 | 22.6 | 0.8161 | 2000 | 67.0 | 23.5 | 0.8628 | 2010 | 70.7 | 24.6 | 0.9066 |
| Sweden | 1935 | 63.7 | 24.2 | 0.7975 | 1943 | 67.3 | 25.4 | 0.8430 | 1949 | 69.5 | 25.1 | 0.8875 |
| Sweden | 1936 | 63.4 | 23.9 | 0.7981 | 1944 | 66.2 | 24.8 | 0.8330 | 1950 | 69.9 | 25.1 | 0.8942 |
| Sweden | 1937 | 63.4 | 23.9 | 0.7980 | 1945 | 67.2 | 25.1 | 0.8454 | 1951 | 70.0 | 25.3 | 0.8954 |
| Sweden | 1938 | 64.0 | 24.0 | 0.8060 | - | - | - | - | 1952 | 70.4 | 25.4 | 0.8994 |
| Sweden | - | - | - | - | - | - | - | - | 1953 | 70.4 | 25.4 | 0.8991 |
| Sweden | - | - | - | - | - | - | - | - | 1954 | 70.9 | 25.6 | 0.9048 |
| Italy | 1949 | 62.6 | 24.0 | 0.7944 | 1958 | 66.5 | 24.3 | 0.8555 | - | - | - | - |
| Italy | 1950 | 62.6 | 24.0 | 0.8134 | 1959 | 66.9 | 24.5 | 0.8588 | - | - | - | - |
| Italy | 1951 | 64.0 | 24.2 | 0.8243 | 1960 | 66.7 | 24.0 | 0.8599 | - | - | - | - |
| Italy | 1952 | 63.5 | 23.6 | 0.8342 | 1961 | 67.2 | 24.3 | 0.8625 | - | - | - | - |
| Italy | - | - | - | - | 1962 | 66.5 | 23.7 | 0.8609 | - | - | - | - |
| Italy | - | - | - | - | 1963 | 66.6 | 23.7 | 0.8602 | - | - | - | - |
| Italy | - | - | - | - | 1964 | 67.6 | 24.2 | 0.8653 | - | - | - | - |
| Japan | 1954 | 62.5 | 22.1 | 0.8037 | 1962 | 66.2 | 22.5 | 0.8650 | 1971 | 70.1 | 24.6 | 0.9017 |
| Japan | 1955 | 63.6 | 22.4 | 0.8202 | 1963 | 67.2 | 23.1 | 0.8725 | 1972 | 70.5 | 24.9 | 0.9038 |
| Japan | 1956 | 63.5 | 22.0 | 0.8235 | - | - | - | - | 1973 | 70.8 | 25.0 | 0.9067 |
| Japan | 1957 | 63.3 | 21.6 | 0.8261 | - | - | - | - | - | - | - | - |
| Japan | 1957 | 64.8 | 22.6 | 0.8391 | - | - | - | - | - | - | - | - |
| Switzerland | 1941 | 62.8 | 22.2 | 0.8073 | 1950 | 66.7 | 23.4 | 0.8617 | 1967 | 69.7 | 24.4 | 0.9009 |
| Switzerland | 1942 | 63.4 | 22.5 | 0.8149 | 1951 | 66.3 | 23.1 | 0.8594 | 1969 | 69.7 | 24.3 | 0.9017 |
| Switzerland | 1943 | 63.5 | 22.7 | 0.8146 | 1952 | 67.2 | 23.7 | 0.8688 | 1970 | 70.0 | 24.6 | 0.9021 |
| Switzerland | 1944 | 62.6 | 22.0 | 0.8076 | 1953 | 67.1 | 23.4 | 0.8693 | 1971 | 70.2 | 24.7 | 0.9034 |
| Switzerland | 1945 | 63.3 | 22.4 | 0.8155 | - | - | - | - | 1972 | 70.6 | 25.0 | 0.9070 |
| Switzerland | 1946 | 64.2 | 22.8 | 0.8272 | - | - | - | - | - | - | - | - |
| Switzerland | 1947 | 64.1 | 22.7 | 0.8248 | - | - | - | - | - | - | - | - |
| Portugal | 1965 | 62.9 | 23.2 | 0.8092 | 1977 | 66.7 | 23.6 | 0.8559 | - | - | - | - |
| Portugal | 1966 | 62.4 | 22.6 | 0.8059 | 1978 | 67.3 | 24.0 | 0.8615 | - | - | - | - |
| Portugal | 1967 | 63.2 | 23.0 | 0.8123 | - | - | - | - | - | - | - | - |
| Portugal | 1968 | 63.6 | 23.3 | 0.8187 | - | - | - | - | - | - | - | - |
| Portugal | 1969 | 63.2 | 22.4 | 0.8184 | - | - | - | - | - | - | - | - |
| Portugal | 1970 | 63.9 | 23.4 | 0.8221 | - | - | - | - | - | - | - | - |
| Portugal | 1971 | 63.5 | 22.8 | 0.8189 | - | - | - | - | - | - | - | - |
| UK | 1945 | 62.8 | 22.7 | 0.8016 | 1950 | 66.2 | 22.4 | 0.8689 | 1976 | 69.5 | 23.3 | 0.9151 |
| UK | 1946 | 63.8 | 22.7 | 0.8222 | 1952 | 66.8 | 22.5 | 0.8802 | 1977 | 70.0 | 23.7 | 0.9176 |
| UK | 1947 | 63.9 | 22.3 | 0.8304 | 1953 | 67.1 | 22.5 | 0.8851 | 1979 | 70.1 | 23.6 | 0.9189 |
| UK | - | - | - | - | - | - | - | - | 1980 | 70.5 | 23.9 | 0.9229 |
| UK | - | - | - | - | - | - | - | - | 1981 | 70.9 | 24.1 | 0.9254 |
| Finland | 1951 | 62.3 | 20.6 | 0.8070 | 1972 | 66.6 | 22.0 | 0.8636 | 1983 | 70.2 | 23.8 | 0.9100 |
| Finland | 1952 | 63.4 | 21.0 | 0.8233 | 1973 | 67.0 | 22.0 | 0.8710 | 1984 | 70.5 | 24.1 | 0.9097 |
| Finland | 1953 | 63.2 | 20.9 | 0.8265 | 1974 | 66.9 | 22.0 | 0.8697 | 1987 | 70.7 | 24.4 | 0.9065 |
| Finland | 1954 | 64.2 | 21.4 | 0.8379 | 1975 | 67.4 | 22.3 | 0.8753 | - | - | - | - |
| Finland | 1955 | 63.8 | 21.0 | 0.8356 | - | - | - | - | - | - | - | - |
| Finland | 1956 | 64.6 | 21.4 | 0.8402 | - | - | - | - | - | - | - | - |
| Denmark | 1936 | 62.4 | 23.6 | 0.7955 | 1946 | 66.3 | 24.6 | 0.8445 | 1954 | 69.9 | 25.3 | 0.8951 |
| Denmark | 1937 | 62.8 | 23.5 | 0.8052 | 1947 | 67.4 | 24.8 | 0.8601 | 1956 | 70.4 | 25.4 | 0.9023 |
| Denmark | 1938 | 63.9 | 23.9 | 0.8157 | - | - | - | - | 1958 | 70.5 | 25.2 | 0.9060 |
| Denmark | - | - | - | - | - | - | - | - | 1959 | 70.6 | 25.4 | 0.9040 |
| Denmark | - | - | - | - | - | - | - | - | 1968 | 70.8 | 25.0 | 0.9106 |
| Belgium | 1949 | 62.9 | 22.5 | 0.8085 | 1957 | 66.6 | 23.1 | 0.8677 | 1980 | 69.9 | 24.0 | 0.9073 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Belgium | - | - | - | - | 1958 | 67.3 | 23.3 | 0.8768 | 1981 | 70.3 | 24.2 | 0.9106 |
| Belgium | - | - | - | - | - | - | - | - | 1982 | 70.6 | 24.4 | 0.9145 |
| Belgium | - | - | - | - | - | - | - | - | 1983 | 70.6 | 24.4 | 0.9129 |
| Belgium | - | - | - | - | - | - | - | - | 1984 | 71.0 | 24.7 | 0.9156 |

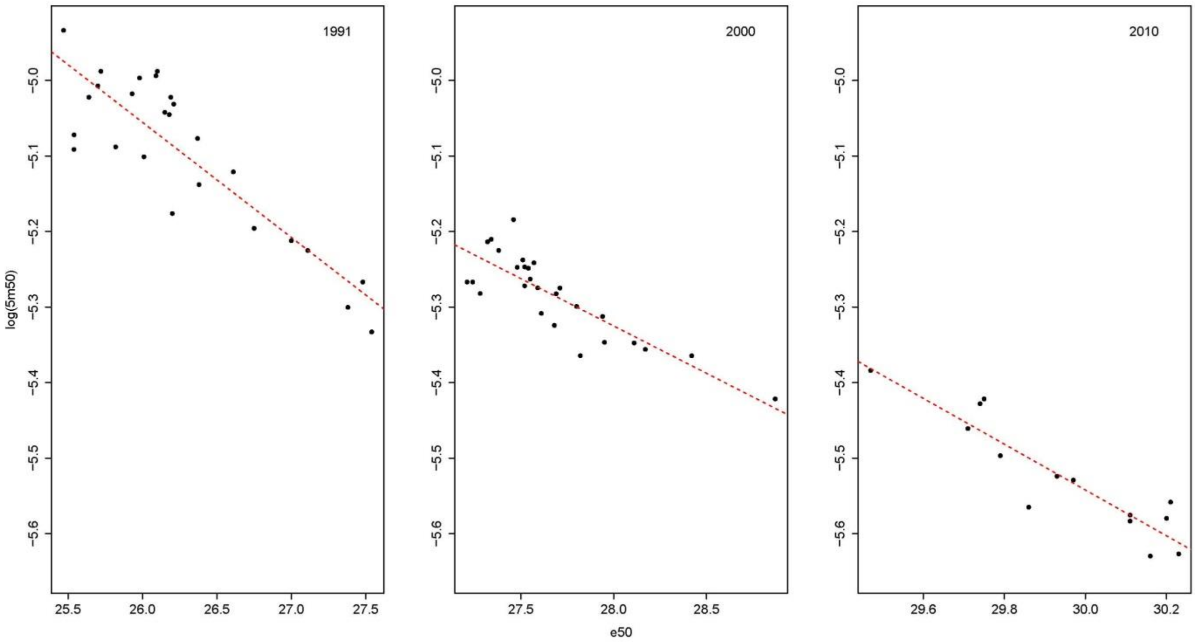
Source: HMD data.

b) Choosing the mortality pattern for 1991, 2000 and 2010

Aside from the question of mortality levels, it is also necessary to make assumptions about the relationships that determine the age pattern of mortality in Brazil. In the Gompertz-Perks model, the mortality curve is fully specified only when a chosen value of $e_{50}$ is accompanied by assumptions regarding ${}_5m_{50}$, $c$ and $v$. The parameters $e_{50}$, ${}_5m_{50}$ and $c$ tend to be strongly correlated, so they must be chosen in a manner to insure that the resulting mortality curve is plausible. Therefore, the relationships between these parameters are expressed here by a series of regression, based on life tables indicated on Tables 1 and 2.

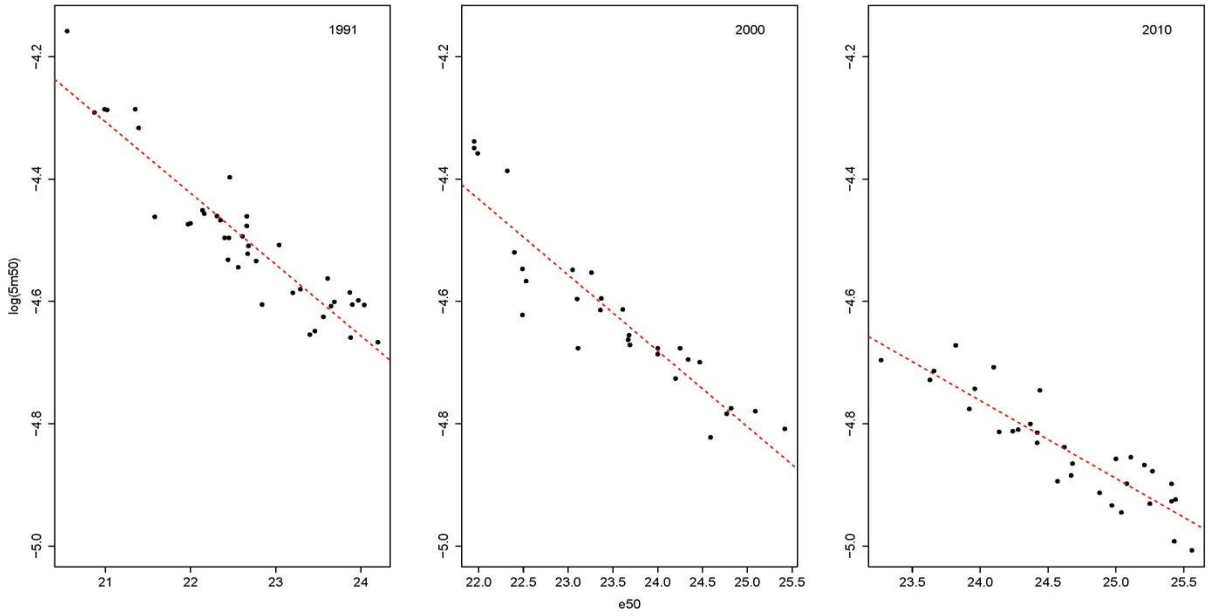
In the first regression we choose ${}_5m_{50}$ for a given level of $e_{50}$. Figures 1 and 2 present the inverse log-linear relationship that is typical for $e_{50}$ and ${}_5m_{50}$, for females and males respectively in 1991, 2000 and 2010. These figures also show the regression line. In the analyses that follow, the choice of ${}_5m_{50}$ for a given $e_{50}$ is centered on the value given by these regression lines. The regressions of $\log({}_5m_{50})$ on $e_{50}$ for females explain 76% of the variance in $\log({}_5m_{50})$ in 1991, 71% in 2000 and 84% in 2010. For males these percentage are 86%, 84% and 81% for 1991, 2000 and 2010, respectively. Tables 1A and 2A in the Appendix present the results of all regressions to predict $\log({}_5m_{50})$ (Model 1) in 1991, 2000 and 2010, for females and males respectively.

**Figure 1** – Relationship between $e_{50}$ and ${}_5m_{50}$, females, 1991, 2000, 2010.




Note: The dashed line corresponds to the regression line.
Source: authors' own calculation based on HMD data.

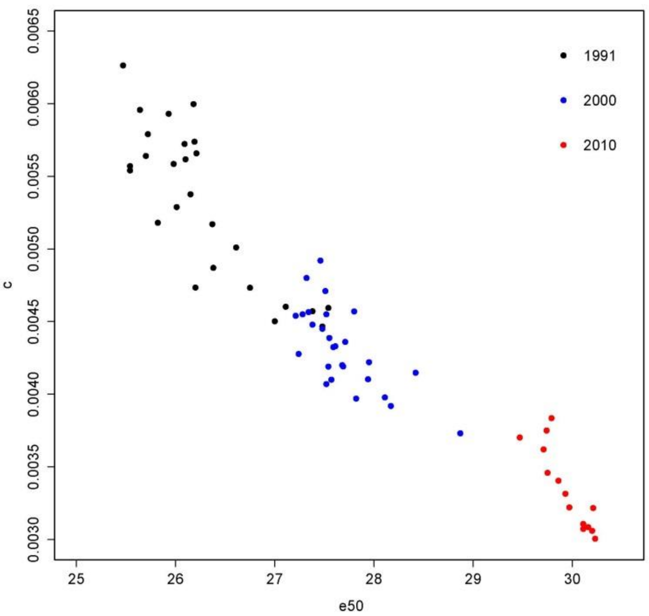
**Figure 2** – Relationship between $e_{50}$ and ${}_5m_{50}$, males, 1991, 2000, 2010.




Note: The dashed line corresponds to the regression line.
Source: authors' own calculation based on HMD data.

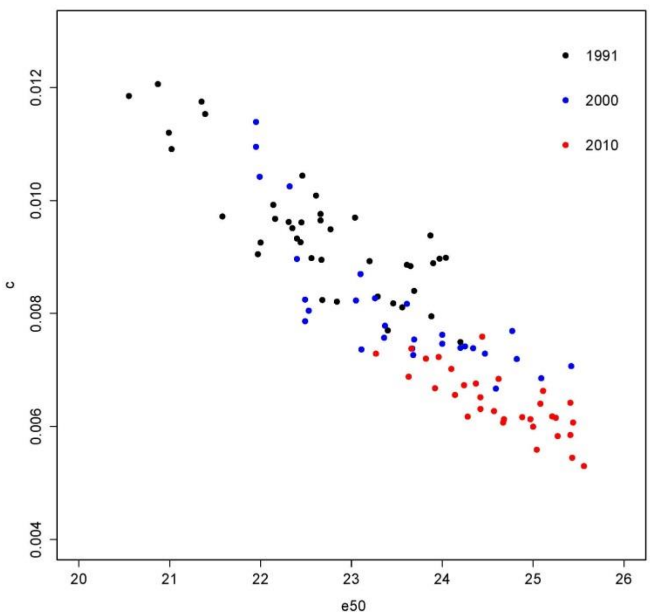
The method for choosing values of the background mortality parameter, *c*, involves a multiple regression. Figures 3 and 4 present the strong relationship between *c* and $e_{50}$ in 1991, 2000 and 2010, for females and males respectively. For all years analyzed the Spearman's correlation coefficient are negative and higher than 0.70 for females, and higher than 0.78 males.

**Figure 3 -** Relationship between $e_{50}$ and $c$, females, 1991, 2000, 2010.




Source: authors' own calculation based on HMD 2016.

**Figure 4 -** Relationship between $e_{50}$ and $c$, males, 1991, 2000, 2010.

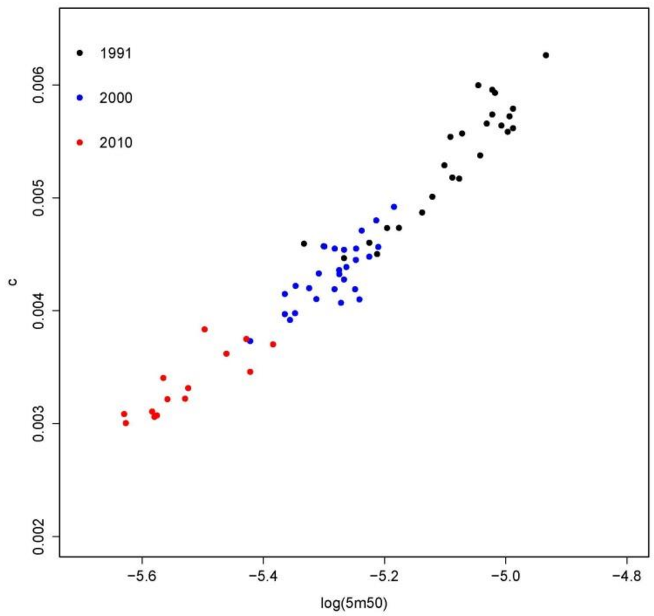



Source: authors' own calculation based on HMD 2016.

The relationship between $c$ and ${}_5m_{50}$ are presented in Figures 5 and 6. A strong relationship is also observed between ${}_5m_{50}$ and $c$. The Spearman's correlation coefficient is 0.89 for females and 0.91 for males in 1991, 0.71 for females and 0.89 for males in 2000, and 0.85 for females and 0.86 for males in 2010.

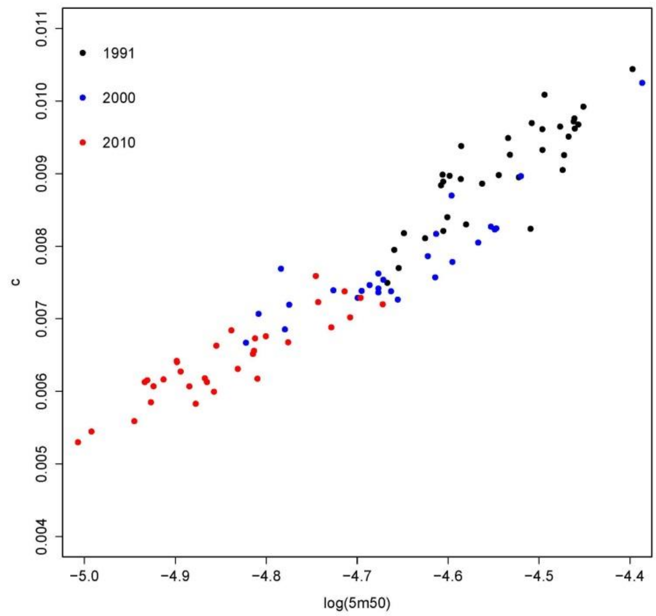
For males in all analyzed years, the best prediction is achieved by regressing $c$ on both $e_{50}$ and ${}_5m_{50}$ (Table 2A-Model 2). For females in 1991 and 2000, the best regression is between $c$ and ${}_5m_{50}$, while for 2010 the best regression is achieved between $c$ and $e_{50}$ (Table 1A-Model 2). The criteria use to select the best regressions are the *t-student* test to check the significance of coefficients, and the coefficient of determination ($R^2$), which measures the variability in the data explained by the regression model.

**Figure 5 -** Relationship between ${}_5m_{50}$ and *c*, females, 1991, 2000, 2010.




Source: authors' own calculation based on HMD 2016.

**Figure 6 -** Relationship between ${}_5m_{50}$ and *c*, males, 1991, 2000, 2010.




Source: authors' own calculation based on HMD 2016.

The last parameter to be chosen is *v*. It determines the upper asymptote of the mortality curve. Unlike the other parameters, the parameter *v* is chosen in a more arbitrary fashion, based on empirical evidence about typical value of this parameter. Robine et al. (2005) and Gampe (2010) revealed that the human mortality after age 110 is flat at a constant level of 0.7 in today's low mortality countries, corresponding an annual probability of death of 0.5. Since higher values of *v* are associated with lower mortality rates at high ages (Wilmoth 1995), we assume a lower value of *v* than that of the low-mortality countries. Thus, we consider in this study $v = 0.6$.

In addition to choose the four parameters of the Gompertz-Perks curve, the proportion surviving from birth to age 50 ($l(50)$) is needed to estimate the prevalence of centenarians. First we tried to choose $l(50)$ using multiple regression of $l(50)$ against $e_{50}$, ${}_5m_{50}$ and *c*. However, these regression were not satisfactory, explaining less than 30% of the variance in $l(50)$, for both female and male. Therefore, we use the same strategy as that to choose the $e_{50}$. We assume that the proportion surviving from birth to age 50 in Brazil is the average of $l(50)$ derived from the listed life tables in Tables 1 and 2, respectively for females and males.

From these results, we define that $l(50)$ is: 0.8917 for female and 0.8161 for male in 1991, 0.9298 for female and 0.8628 for male in 2000, and 0.9516 for female and 0.9066 for male in 2010.

Having chosen all parameters of the Gompertz-Perks curve, we can calculate the probability of survival from age 50 to 100:

$$\frac{l(x)}{l(y)} = \begin{cases} \left(\dfrac{1+vae^{bx}}{1+vae^{by}}\right)^{\frac{1}{b}\left(c-\frac{1}{v}\right)} e^{-c(x-y)} & if\ v>0 \\ e^{\left\{-c(x-y)-\frac{a}{b}\left(e^{bx}-e^{by}\right)\right\}} & if\ v=0 \end{cases} \qquad (2)$$

where $x$ = 100 and $y$ =50.

In this first estimation procedure, we estimate the prevalence of centenarians in a stable population as follows:

$$\begin{aligned} {}_{\omega}c_{100} &= \int_{100}^{\omega} be^{-rx} l(x)dx \\ &= \frac{\int_{100}^{\omega} e^{-rx} l(x)dx}{\int_{0}^{\omega} e^{-rx} l(x)dx} \\ &= \frac{\int_{100}^{\omega} e^{-rx} l(x)dx}{\int_{0}^{y} e^{-rx} l(x)dx + \int_{y}^{\omega} e^{-rx} l(x)dx} \qquad (3) \\ &= \frac{l(y)\int_{100}^{\omega} e^{-rx} \frac{l(x)}{l(y)} dx}{\int_{0}^{y} e^{-rx} l(x)dx + l(y)\int_{y}^{\omega} e^{-rx} \frac{l(x)}{l(y)} dx} \end{aligned}$$

where $x$ in these integrals denotes age, $b$ (only in this equation (3)) is the birth rate, $\omega$ is the latest attained age (110 in this study), $r$ is the constant population growth rate, $l(x)$ is the probability of survival from age 0 to $x$, and $y$ is the age 50.

The idea of the growth rate in formula (3) is to adjust the size of the cohort aged 50 with the size of cohort aged 100 at the same period. For instance, in 2010, the number of centenarians is smaller than the number of 50-years old people not only because some 50-years old died before the age100, but also because the centenarians in 2010 derived from the cohort aged 50 in 1960 that was smaller at age 50 than the cohort aged 50 in 2010 by the factor $e^{-r}$ (Preston et al. 2001). For instance, to estimate the centenarian population in 2010 we use the mean annualized growth rates between 1960 and 2010, for 2000, we use the annual growth rate between 1950 and 2000, and for 1991 we use the annual growth between 1950 and 1990. In the latest case, the growth rate is estimated in a time interval of 40 years due to availability of United Nations data. The choice of using the United Nations data can be justified by the necessary adjustments for deficiencies in data made by the United Nations.

Table 3 presents the growth rates used in the first estimation procedure to calculate the number of centenarians in 1991, 2000 and 2010. All these growth rates are calculated as depicted in Preston et al. (2001).

**Table 3**: Growth rates by sex used in the first estimation procedure, Brazil.

| | **Male** | **Female** |
|---|---|---|
| 1940-1990 | 0.0256 | 0.0256 |
| 1950-2000 | 0.0235 | 0.0237 |
| 1960-2010 | 0.0200 | 0.0204 |

Source: authors' own calculation based on UN data (2015).

c) Simulation

We select the values of parameters in equation (3) in a manner that reflects our uncertainty about their true values in Brazil. In order to evaluate our estimates, we produce a set of simulations where the exact values of the chosen parameters are different for each trial. The first set of simulations is using the selected parameters, named "base model". Next, we make various modifications to the "base model" and we compute additional sets of simulations in order to evaluate the sensitivity of our results to changes in the values of the parameters.

For each trial $e_{50}$ is fixed at the values presented in Tables 1 and 2, for females and males, respectively. The parameters ${}_5m_{50}$, $c$ and $l(50)$ are drawn at random from normal distribution whose means are chosen conditionally based on all previously selected parameters. For instance, the distribution for ${}_5m_{50}$ is centered on the predicted value from a simple regression on $e_{50}$. In each case, the standard error for the normal distribution is set equal to the maximum residual (from the respective regressions) divided by 3. The parameter $v$ is drawn at random from a normal distribution centered on 0.6, with a standard error of 0.2. The population growth rates are fixed at the values presented in Table 3.

After choosing the parameters for each simulation trial, the equation (3) is used to compute the prevalence of centenarians, expressed as a proportion of the total stable population.

*ii. Second estimation procedure*

The second estimation procedure also considers the Gompertz-Perks curve with the same parameters defined for the first procedure. However, unlike the first estimation procedure, this procedure estimates the prevalence of centenarians in a non-stable population, assuming a set of age-specific growth rates as follows:

$$ {}_{\omega}c^{*}_{100} = \frac{l(y)\int_{100}^{\omega} e^{-r_1 x}\frac{l(x)}{l(y)}dx}{\int_{0}^{y} e^{-r_2 x} l(x)dx + l(y)\int_{y}^{\omega} e^{-r_3 x}\frac{l(x)}{l(y)}dx} \qquad (4) $$

where $r_1$ is the growth rate of population age 100 years or over, $r_2$ is the growth rate at ages 0 to 50, and $r_3$ is the growth rate of population age 50 years or over. Due to quality issues in centenarian data, as already mentioned, we assume that $r_1$ is the growth rate of population age 50 years or over.

As in the first procedure, to estimate the number of centenarians in 2010, we use the annual growth rates between 1960 and 2010. For 2000 we use the annual growth rates between 1950 and 2000, and for 1991 the annual growth rates between 1950 and 1990. Table 4 presents the set of age-specific annual growth rates using in the second.

**Table 4**: Age-specific growth rates by sex used in the second estimation procedure, Brazil.

| | **Male** | | | **Female** | | |
|---|---|---|---|---|---|---|
| | **1950-1990** | **1950-2000** | **1960-2010** | **1950-1990** | **1950-2000** | **1960-2010** |
| $r_1$ *(50+)* | 0.0300 | 0.0302 | 0.0318 | 0.0297 | 0.0303 | 0.0322 |
| $r_2$ *(0-50)* | 0.0251 | 0.0226 | 0.0181 | 0.0250 | 0.0226 | 0.0181 |
| $r_3 = r_1$ | 0.0300 | 0.0302 | 0.0318 | 0.0297 | 0.0303 | 0.0322 |

Source: authors' own calculation based on UN data (2015).

It is relevant to mention that we use here the same simulation process describe in the first estimation procedure.

*iii. Third estimation procedure*

The third estimation procedure involves a reconstruction of two cohorts at the same census (one initial cohort aged 50-54 years and the other cohort aged 55-59), advancing these cohorts through successive 5-year time intervals. To do that, we apply the forward-survival method, "projecting" the population at one census date to a later date through the use of appropriate survival probabilities. The accuracy of estimates using this approach depends upon the accuracy of the initial population and the survival rates. In addition, we assume that net immigration equaled or approximated zero. The initial population is the population at age groups 50-54 and 55-59 recorded in the 1940, 1950 and 1960 Brazilian census. The data quality issues common in centenarian data, like age exaggeration, are presumably somewhat more accurate at these younger ages, at least in terms of 5-years age groups. On the other hand, some error in the survival rates, like under-registration of deaths, can overestimate our centenarian population. On balance, the projected number of centenarians in 1990, 2000 and 2010 based on the 1940, 1950 and 1960 census, respectively, are believed to be a more accurate estimate and less likely to be an overstatement than the number of centenarians direct report in census.

Applying survival probabilities (in quinquennial age groups by sex) to the census population aged 50-54 and 55-59 yields an estimate of the number of individuals aged 100-104 and 105-109 years, respectively.

The equations used to reconstruct the cohort aged 50-54 at one census date until the age group 100-104 50 years later are:

$$\begin{aligned} {}_5N_{55}^{\,i+5} &= {}_5N_{50}^{\,i}\left({}_5p_{50}^{\,i}\right) \\ {}_5N_{60}^{\,i+10} &= {}_5N_{55}^{\,i+5}\left({}_5p_{55}^{\,i+5}\right) \\ &\vdots \\ {}_5N_{100}^{\,i+50} &= {}_5N_{95}^{\,i+45}\left({}_5p_{95}^{\,i+45}\right) \end{aligned} \qquad (5)$$

where $i$ is the initial census year, ${}_5N_{50}^{\,i}$ is the initial cohort at age group 50-54 at year $i$, ${}_5N_{55}^{\,i+5}$ is the estimated number of individuals at age group 55-59 at year $i+5$, and so on; ${}_5p_{50}^{\,i}$ is the probability of surviving from age 50 to 54 at year $i$, ${}_5p_{55}^{\,i+5}$ is the probability of surviving from age 55 to 59 at year $i+5$, and so on.

For the cohort aged 55-59 at one census date until the age group 105-109 the equations are:

$$
\begin{aligned}
{}_5N_{60}^{\;i+5} &= {}_5N_{55}^{\;i}\left({}_5p_{55}^{\;i}\right) \\
{}_5N_{65}^{\;i+10} &= {}_5N_{60}^{\;i+5}\left({}_5p_{60}^{\;i+5}\right) \\
&\vdots \\
{}_5N_{105}^{\;i+50} &= {}_5N_{100}^{\;i+45}\left({}_5p_{100}^{\;i+45}\right)
\end{aligned}
\qquad (6)
$$

Through the equations (5) and (6) we achieve the number of individuals at age groups 100-104 and 105-109, respectively for 1991, 2000 and 2010 in Brazil.

The choice of life tables from which the survival rates are computed is crucial to this estimation procedure. Therefore, we use the Brazilian abridged life tables for males and females, calculated by the United Nations from 1950-1955 to 2010-2015 (UN 2015). The United Nations made the necessary adjustments for deficiencies in age reporting, under-enumeration, or underreporting of vital events. The open-ended age interval of these life tables begins with age 85, thus, in order to estimate death rates until the age 110, we fit the Kannisto model of old age mortality to distribute deaths in the open-ended age interval (Thatcher et al. 1998). The Kannisto model has the following form:

$$\mu(x) = \frac{ae^{b(x-x_0)}}{1+ae^{b(x-x_0)}} \qquad (6)$$

where $x_0 = 85-20$, and *a* and *b* are unknown parameters estimated through the optimization of the equation (6).

Tables 6 and 7 show the age-specific survival probabilities for female and male, respectively, used to cohort reconstruction. Due to the availability of survival probabilities only from 1950-1955, we assume that the survival probabilities at age groups 50-54, 55-59 and 60-64 for the periods 1940-1945 and 1945-1950 are equal to the survival probabilities at the same age groups for the period 1950-1955. The size of initial cohorts by sex a presented in Table 8.

**Table 6** – Survival probabilities by 5-year age group, female, Brazil, 1950-1955 to 2005-2010.

| Age Group | 1940-1945 | 1945-1950 | 1950-1955 | 1955-1960 | 1960-1965 | 1965-1970 | 1970-1975 | 1975-1980 | 1980-1985 | 1985-1990 | 1990-1995 | 1995-2000 | 2000-2005 | 2005-2010 | 2010-2015 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 50-54 | 0.9287 | | 0.9287 | 0.9342 | 0.9421 | | | | | | | | | | |
| 55-59 | 0.9043 | 0.9043 | 0.9043 | 0.9112 | 0.9209 | 0.9283 | | | | | | | | | |
| 60-64 | | 0.8674 | 0.8674 | 0.8765 | 0.8883 | 0.8961 | 0.9036 | | | | | | | | |
| 65-69 | | | 0.8133 | 0.8252 | 0.8396 | 0.8466 | 0.8543 | 0.8593 | | | | | | | |
| 70-74 | | | | 0.7523 | 0.7692 | 0.7731 | 0.7793 | 0.7853 | 0.7921 | | | | | | |
| 75-79 | | | | | 0.6593 | 0.6602 | 0.6642 | 0.6690 | 0.6742 | 0.6855 | | | | | |
| 80-84 | | | | | | 0.4849 | 0.4954 | 0.5085 | 0.5234 | 0.5599 | 0.5694 | | | | |
| 85-89 | | | | | | | 0.3623 | 0.3838 | 0.3924 | 0.4280 | 0.4490 | 0.5038 | | | |
| 90-94 | | | | | | | | 0.2585 | 0.2733 | 0.3069 | 0.3327 | 0.3907 | 0.4403 | | |
| 95-99 | | | | | | | | | 0.1531 | 0.1843 | 0.2060 | 0.2629 | 0.2789 | 0.2990 | |
| 100-104 | | | | | | | | | | 0.0430 | 0.0607 | 0.0779 | 0.0806 | 0.0990 | 0.1097 |

Source: authors' own calculation based on UN data (2015).

**Table 7** – Survival probabilities by 5-year age group, male, Brazil, 1950-1955 to 2005-2010.

| Age Group | 1940-1945 | 1945-1950 | 1950-1955 | 1955-1960 | 1960-1965 | 1965-1970 | 1970-1975 | 1975-1980 | 1980-1985 | 1985-1990 | 1990-1995 | 1995-2000 | 2000-2005 | 2005-2010 | 2010-2015 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 50-54 | 0.9165 | | 0.9165 | 0.9239 | 0.9299 | | | | | | | | | | |
| 55-59 | 0.8868 | 0.8868 | 0.8868 | 0.8949 | 0.9014 | 0.9072 | | | | | | | | | |
| 60-64 | | 0.8405 | 0.8405 | 0.8503 | 0.8578 | 0.8649 | 0.8672 | | | | | | | | |
| 65-69 | | | 0.7768 | 0.7882 | 0.7960 | 0.8040 | 0.8059 | 0.7941 | | | | | | | |
| 70-74 | | | | 0.6982 | 0.7074 | 0.7174 | 0.7193 | 0.7027 | 0.6926 | | | | | | |
| 75-79 | | | | | 0.5880 | 0.6015 | 0.6044 | 0.5829 | 0.5708 | 0.5778 | | | | | |
| 80-84 | | | | | | 0.4581 | 0.4600 | 0.4608 | 0.4675 | 0.4709 | 0.4817 | | | | |
| 85-89 | | | | | | | 0.3189 | 0.3434 | 0.3446 | 0.3413 | 0.3617 | 0.4179 | | | |
| 90-94 | | | | | | | | 0.2300 | 0.2459 | 0.2567 | 0.2635 | 0.3080 | 0.3520 | | |
| 95-99 | | | | | | | | | 0.1234 | 0.1464 | 0.1608 | 0.2073 | 0.2356 | 0.2489 | |
| 100-104 | | | | | | | | | | 0.0401 | 0.0588 | 0.0661 | 0.0693 | 0.0732 | 0.0879 |

Source: authors' own calculation based on UN data (2015).

**Table 8** – Initial cohorts at age groups 50-54 and 55-59 by sex, Brazil

| Year | Age group 50-54 | | Age group 55-59 | |
|---|---|---|---|---|
| | Female | Male | Female | Male |
| 1940 | 549,886 | 590,975 | 441,568 | 462,478 |
| 1950 | 715,320 | 763,270 | 574,414 | 597,310 |
| 1960 | 1,017,706 | 1,094,744 | 817,234 | 856,710 |

Source: Census Data 1940, 1950 and 1960.

Through this estimation procedure we reconstruct Brazilian cohorts at age groups 50-54 and 55-59 to obtain the number of centenarians in 1990, 2000 and 2010.

## Results

Through three different methods we estimate the number of centenarians by sex in 1991, 2000 and 2010 in Brazil, as presented in Table 9. All methods suggest that the number of people celebrating their 100th birthday multiplied several fold in the last decades in Brazil. Moreover, all estimates clearly reflect the considerable overstatement of centenarians in each of the past three censuses.

**Table 9**: Number of centenarians, recorded and estimated, by sex, Brazil, 1991, 2000 and 2010.

| Estimation Method | Census Count | | | Wilmoth's Method Stable Population | | | Wilmoth's Method Non-Stable Population | | | Forward-Survival Method Cohort Reconstruction | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **1991** | **2000** | **2010** | **1991** | **2000** | **2010** | **1991** | **2000** | **2010** | **1990** | **2000** | **2010** |
| Male | 4,382 | 10,423 | 7,247 | 2,774 | 3,632 | 5,919 | 1,798 | 2,158 | 4,793 | 1,226 | 1,645 | 5,116 |
| Female | 8,914 | 14,153 | 16,989 | 4,701 | 6,335 | 15,390 | 2,808 | 5,171 | 14,656 | 2,582 | 4,842 | 15,001 |
| **Total** | **13,296** | **24,576** | **24,236** | **7,475** | **9,967** | **21,310** | **4,607** | **7,328** | **19,449** | **3,808** | **6,487** | **20,117** |

Source: authors' own calculation, and 1991, 2000 and 2010 Census.

The estimated number of centenarians falls approximately in the range 4,000-7,500 in 1991, in the range 6,500-10,000 in 2000, and in the range 19,500-21,300 in 2010. Two of the estimates (Wilmoth's method for non-stable population, and the forward-survival method) are grouped at the lower end of the ranges. The largest estimates derived from the Wilmoth's method assuming a stable population. Note that our estimates for 2010 are the closest to the number of centenarians recorded in the 2010 census, suggesting progress in census data quality.

In order to provide further support for our estimates derived from both Wilmoth's methods, Tables 10 and 11 present the resulting distribution of the number of centenarians from simulation trials, respectively for a stable and non-stable population. The range of estimates reflects the uncertainty about the number of centenarians that results from our uncertainty about the relationship between the various parameters of the mortality model. Note that the median number of centenarians derived from the Wilmoth's method assuming a non-stable population (Table 11) is very close to estimates derived from the forward-survival method in all analyzed years (Table 9). To complement our results, the sensitivity analysis of the centenarian prevalence estimates is evaluated in Tables 3A and 4A in the Appendix. This analysis varies the levels of all parameters. This sensitivity tests do not alter the most important conclusion of our analysis that the number of centenarians rapidly increased in the last decades. Not surprisingly, the overall mortality level has the largest impact in our predictions regarding the number of centenarians through the last decades in Brazil. In addition, the uncertainty about the growth rates does not affect our conclusion.

**Table 10:** Simulated estimates of the number of centenarians assuming a stable population, by sex in Brazil, 1991, 2000 and 2010.

| | 1991 | | | 2000 | | | 2010 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Percentile | | | Percentile | | | Percentile | | |
| | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 |
| Male | 2,040 | 2,752 | 3,620 | 2,406 | 3,596 | 5,070 | 3,974 | 5,932 | 8,148 |
| Female | 2,196 | 4,145 | 6,628 | 3,597 | 6,237 | 9,479 | 9,671 | 15,280 | 21,810 |

Source: authors' own calculation.

**Table 11:** Simulated estimates of the number of centenarians assuming a non-stable population, by sex in Brazil, 1991, 2000 and 2010.

| | 1991 | | | 2000 | | | 2010 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Percentile | | | Percentile | | | Percentile | | |
| | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 |
| Male | 1,321 | 1,784 | 2,350 | 1,421 | 2,136 | 3,028 | 3,125 | 4,805 | 6,834 |
| Female | 1,468 | 2,780 | 4,459 | 2,875 | 5,086 | 7,857 | 8,527 | 14,540 | 22,020 |

Source: authors' own calculation.

Women outnumber men in every age group due to higher life expectancy and this is particularly striking in older ages (Tommassini 2005). However, as the ratio of female to male centenarians has begun to fall in recent years as a result of improvements in male mortality.

Analysis by sex of our estimates reveal, as expected, that the number of female centenarians is higher than male over all analyzed years, and through all methods. In addition, the female centenarian population grew more rapidly than the male population between 1991 and 2010. This analysis also suggests that the problem of age exaggeration is not entirely independent of sex. The 2000 census shows 24,576 centenarians; of these 42% were male. Surprisingly, this male proportion of centenarians is too high due to the known female survival advantage at older ages. However, our estimates reveal that on average 30% of centenarians were male in 2000. This result also suggests the accuracy of our estimates regarding the number of centenarians in the 2000 Census.

**Table 12**: Proportion of centenarians in total population (per 10,000) by sex, 1991, 2000 and 2010.

| Country | **1991** | | | **2000** | | | **2010** | | |
|---|---|---|---|---|---|---|---|---|---|
| | Male | Female | Total | Male | Female | Total | Male | Female | Total |
| Brazil (Stable Population) | 0.383 | 0.632 | 0.509 | 0.435 | 0.735 | 0.587 | 0.634 | 1.581 | 1.117 |
| Brazil (Non-Stable Population) | 0.248 | 0.378 | 0.314 | 0.258 | 0.600 | 0.432 | 0.513 | 1.506 | 1.020 |
| Brazil (Cohort Reconstruction) | 0.169 | 0.347 | 0.259 | 0.197 | 0.562 | 0.382 | 0.548 | 1.541 | 1.055 |
| Brazil (Gomes and Turra 2009) | 0.181 | 0.421 | 0.302 | - | - | - | - | - | - |
| Brazil (census bureau) | 0.605 | 1.199 | 0.906 | 1.247 | 1.641 | 1.447 | 0.776 | 1.745 | 1.271 |
| Sweden | 0.278 | 1.097 | 0.693 | 0.322 | 1.681 | 1.008 | 0.523 | 2.897 | 1.715 |
| Japan | 0.109 | 0.419 | 0.267 | 0.325 | 1.469 | 0.909 | 0.912 | 5.629 | 3.330 |
| UK | 0.172 | 1.297 | 0.750 | 0.214 | 1.890 | 1.073 | 0.457 | 3.013 | 1.759 |
| France | 0.191 | 1.255 | 0.737 | 0.323 | 2.367 | 1.375 | 0.680 | 4.590 | 2.696 |

Source: authors' own calculation, 1991, 2000 and 2010 Brazilian Census, and HMD.

Another way to check the accuracy of our estimates is comparing them with the proportion of centenarians of countries with presumably higher data quality, as presented in Table 12. Not surprisingly, analyses by sex reveal higher proportions of centenarians among women than men in all selected countries, including in our estimates to Brazil, in the period 1991 to 2010.

In 1991, the female centenarian share of population estimated through all methods is lower than that of the selected low-mortality countries, except Japan. Only the Wilmoth's method assuming a stable population presents higher proportion of female centenarians than Japan in 1991. For males, only the proportion of centenarians using the forward-survival method is lower than that of the selected low-mortality countries, except Japan. Not surprisingly, the proportion of centenarians for both male and female recorded in the 1991 census is higher than proportions of all selected low-mortality countries.

Comparing our results with the estimates of Gomes and Turra (2009) in 1991, we note similarities. Our proportions of centenarians derived from the Wilmoth's method assuming a non-stable population and from the forward-survival method are close to Gomes and Turra (2009) estimates (Table 12). This similarity provides further support for the accuracy of both results. Moreover, it suggests that our estimates derived from the Wilmoth's method assuming a non-stable population, and from the forward-survival method seem to be more accurate than that of the first estimation procedure in 1991.

In 2000 and 2010, the female centenarian share of population through the Wilmoth's method for non-stable population and using the forward-survival method are lower than that of the low-mortality countries in 2000 and 2010 (Table 12). For males, only the proportion of centenarians using the forward-survival method is lower than that of the low-mortality countries in 2000. In 2010 the male proportion of centenarians from the Wilmoth's method for non-stable population are lower than that of the low-mortality countries, except the UK. As observed in 1991, the estimates from the Wilmoth's method for non-stable population and from the forward-survival method seem to be more accurate than that of the first estimation in 2000 and 2010.

## Conclusion and Discussion

The results suggest the proliferation of centenarians in the last decades in Brazil. Our preferred estimates of the number of centenarians for the past three census years are that derived from the Wilmoth's method assuming a non-stable population and from the forward-survival method used to cohort reconstruction. The Wilmoth's method assuming a non-stable population is more complex than the forward-survival method, however, it has the advantage of the simulation and the sensitivity analysis. Conversely, the forward-survival method is simple but involves some assumptions and depends upon the accuracy of the survival probabilities.

The results revealed that the largest estimates of the number of centenarians derived from the Wilmoth's method assuming a stable population. This method assumes a stable population in Brazil during the period 1991-2010, considering that the life table of a population is constant over time. This assumption is too strong due to the deep demographic changes the Brazilian population has undergone since the 1940s (Carvalho 1974).

The second estimation procedure (Wilmoth's method assuming a non-stable population) does not assume a stable population, considering a set of age-specific growth rates. This procedure is more realistic due to the population demographic changes over the past decades in Brazil. As expected the number of centenarians derived from this procedure is more consistent than the first procedure when both are compared with countries with presumably better data quality. The limitation of this procedure is the uncertainty about the fit of the Gompertz-Perks curve to the mortality pattern at the oldest ages in Brazil, and the uncertainty about the relationships between the various parameters of the mortality model. However, the simulation and the sensitivity analysis provide stronger support to the accuracy of these estimates.

The forward-survival method used to cohort reconstruction depends to the accuracy of the initial population at age groups 50-54 and 55-59 recorded in the 1940, 1950 and 1960 Brazilian census. Nevertheless, the use of quinquennial age groups reduces common errors of reporting ages, for instance the digit preference. Moreover, age statements in the census are presumably more accurate at these younger age groups than for centenarians. Another important issue to the accuracy of the estimates using the forward-survival method is the accuracy of survival rates. As already mentioned, we chose the life tables estimated by the United Nations (2015). Before estimate the survival rates, the United Nations made the necessary adjustments for deficiencies in age reporting, under-enumeration, or underreporting of vital events. The assumption of negligible international migration at age 50 and over is believed to not affect the accuracy of the estimates, due to low migration rates at older ages.

Although even the acceptable estimate of centenarians in 1991, 2000 and 2010 are subject to error, they do provide a range of figures which may reasonably be expected to bracket the true number. Thus, we expect that the true number of centenarians in Brazil falls in the range

3,808-4,607 in 1991, 6,487-7,328 in 2000, and 19,449-20,117 in 2010. These numbers of centenarians is around 1/3 the number recorded in the 1991 census, ¼ the number recorded in the 2000 census, and 4/5 the number recorded in the 2010 census. This comparison between estimated and recorded centenarian population reveals the poor data quality in the 2000 Census and the progress in census data quality between 2000 and 2010. The 2000 census presented the largest overstatement of the number of centenarians among the last three census years.

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## Appendix I

**Table 1A** – Regression results for females, 1991, 2000 and 2010.

| Variables/Intercept | Model 1: Predicting $\log({}_5m_{50})$ | | | Model 2: Predicting *c* | | |
|---|---|---|---|---|---|---|
| | Coef. | Std. Error | p-value | Coef. | Std. Error | p-value |
| **1991** | | | | | | |
| **Intercept** | -1.093 | 0.460 | 0.026 | 0.029 | 0.002 | 0.000 |
| $\mathbf{e_{50}}$ | -0.152 | 0.018 | 0.000 | - | - | - |
| $\mathbf{\log({}_5m_{50})}$ | - | - | - | 0.005 | 0.000 | 0.000 |
| *Number of observations:* | | *26* | | *Number of observations:* | | *26* |
| $R^2$: | | *76%* | | $R^2$: | | *85%* |
| *Test F (p-value):* | | *0.00* | | *Test F (p-value):* | | *0.00* |
| **2000** | | | | | | |
| **Intercept** | -1.817 | 0.438 | 0.000 | 0.026 | 0.003 | 0.000 |
| $\mathbf{e_{50}}$ | -0.125 | 0.016 | 0.000 | - | - | - |
| $\mathbf{\log({}_5m_{50})}$ | - | - | - | 0.004 | 0.001 | 0.000 |
| *Number of observations:* | | *27* | | *Number of observations:* | | *27* |
| $R^2$: | | *71%* | | $R^2$: | | *64%* |
| *Test F (p-value):* | | *0* | | *Test F (p-value):* | | *0* |
| **2010** | | | | | | |
| **Intercept** | 3.558 | 1.148 | 0.009 | 0.036 | 0.005 | 0.000 |
| $\mathbf{e_{50}}$ | -0.303 | 0.038 | 0.000 | -0.001 | 0.000 | 0.000 |
| $\mathbf{\log({}_5m_{50})}$ | - | - | - | - | - | - |
| *Number of observations:* | | *14* | | *Number of observations:* | | *14* |
| $R^2$: | | *84%* | | $R^2$: | | *79%* |
| *Test F (p-value):* | | *0* | | *Test F (p-value):* | | *0* |

Source: Authors' own calculation based on HMD data.

**Table 2A** – Regression results for males, 1991, 2000 and 2010.

| Variables/Intercept | Model 1: Predicting $\log({}_5m_{50})$ Coef. | Std. Error | p-value | Model 2: Predicting *c* Coef. | Std. Error | p-value |
|---|---|---|---|---|---|---|
| **1991** | | | | | | |
| **Intercept** | -1.858 | 0.170 | 0.000 | 0.053 | 0.003 | 0.000 |
| $e_{50}$ | -0.117 | 0.007 | 0.000 | 0.000 | 0.000 | 0.043 |
| $\log({}_5m_{50})$ | - | - | - | 0.012 | 0.001 | 0.000 |
| *Number of observations:* | | *40* | | *Number of observations:* | | *40* |
| $R^2$: | | *86%* | | $R^2$: | | *89%* |
| *Test F (p-value):* | | *0* | | *Test F (p-value):* | | *0* |
| **2000** | Model 1: Predicting $\log({}_5m_{50})$ | | | Model 2: Predicting ${}_1m_{50}$ | | |
| Variables/Intercept | Coef. | Std. Error | p-value | Coef. | Std. Error | p-value |
| **Intercept** | -1.705 | 0.245 | 0.000 | 0.0506 | 0.0030 | 0.0000 |
| $e_{50}$ | -0.124 | 0.010 | 0.000 | 0.0004 | 0.0002 | 0.0346 |
| $\log({}_5m_{50})$ | - | - | - | 0.0114 | 0.0014 | 0.0000 |
| *Number of observations:* | | *29* | | *Number of observations:* | | *29* |
| $R^2$: | | *84%* | | $R^2$: | | *90%* |
| *Test F (p-value):* | | *0* | | *Test F (p-value):* | | *0* |
| **2010** | Model 1: Predicting $\log({}_5m_{50})$ | | | Model 2: Predicting ${}_1m_{50}$ | | |
| | Coef. | Std. Error | p-value | Coef. | Std. Error | p-value |
| **Intercept** | -1.709 | 0.276 | 0.000 | 0.0353 | 0.0027 | 0.0000 |
| $e_{50}$ | -0.127 | 0.011 | 0.000 | 0.0001 | 0.0002 | 0.1100 |
| $\log({}_5m_{50})$ | - | - | - | 0.0066 | 0.0012 | 0.0000 |
| *Number of observations:* | | *32* | | *Number of observations:* | | *32* |
| $R^2$: | | *81%* | | $R^2$: | | *81%* |
| *Test F (p-value):* | | *0* | | *Test F (p-value):* | | *0* |

Source: Authors' own calculation based on HMD data.

**Table 3A** – Sensitivity of the estimated number of centenarians by sex through the Wilmoth's method assuming a stable population, Brazil.

| | Female | | | | | | | | | Male | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1991 | | | 2000 | | | 2010 | | | 1991 | | | 2000 | | | 2010 | | |
| | Percentile | | | Percentile | | | Percentile | | | Percentile | | | Percentile | | | Percentile | | |
| | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 |
| Base Model | 2,196 | 4,145 | 6,628 | 3,597 | 6,237 | 9,479 | 9,671 | 15,280 | 21,810 | 2,040 | 2,752 | 3,620 | 2,406 | 3,596 | 5,070 | 3,974 | 5,932 | 8,148 |
| $e_{50}$ = + 1 year | 2,797 | 4,968 | 7,584 | 4,523 | 7,430 | 10,840 | 11,990 | 18,110 | 24,940 | 2,611 | 3,423 | 4,384 | 3,055 | 4,396 | 5,997 | 4,976 | 7,143 | 9,509 |
| $e_{50}$ = - 1 year | 1,726 | 3,461 | 5,796 | 2,864 | 5,234 | 8,288 | 7,808 | 12,900 | 19,070 | 1,595 | 2,215 | 2,990 | 1,896 | 2,944 | 4,287 | 3,177 | 4,930 | 6,982 |
| ${}_5m_{50}$ = +10% | 1,803 | 3,792 | 6,473 | 3,065 | 5,805 | 9,315 | 8,559 | 14,480 | 21,590 | 1,539 | 2,214 | 3,075 | 1,892 | 3,060 | 4,580 | 3,287 | 5,279 | 7,623 |
| ${}_5m_{50}$ = -10% | 2,638 | 4,514 | 6,788 | 4,181 | 6,681 | 9,638 | 10,860 | 16,100 | 22,030 | 2,630 | 2,254 | 3,469 | 2,994 | 4,176 | 5,579 | 4,735 | 6,623 | 8,685 |
| *c* = +10% | 2,139 | 4,040 | 6,460 | 3,523 | 6,105 | 9,281 | 9,517 | 15,040 | 21,460 | 1,940 | 2,618 | 3,443 | 2,312 | 3,457 | 4,876 | 3,854 | 5,753 | 7,902 |
| *c* = -10% | 2,255 | 4,253 | 6,804 | 3,675 | 6,368 | 9,677 | 9,829 | 15,530 | 21,170 | 2,144 | 2,893 | 3,805 | 2,504 | 3,740 | 5,270 | 4,098 | 6,118 | 8,401 |
| *v* = 0.7 | 2,227 | 4,178 | 6,657 | 3,642 | 6,280 | 9,521 | 9,774 | 15,380 | 21,900 | 2,073 | 2,786 | 3,655 | 2,441 | 3,633 | 5,107 | 4,024 | 5,984 | 8,198 |
| *v* = 0.5 | 2,166 | 4,113 | 6,598 | 3,553 | 6,189 | 9,437 | 9,569 | 15,180 | 21,720 | 2,008 | 2,718 | 3,584 | 2,371 | 3,558 | 5,032 | 3,924 | 5,881 | 8,097 |
| $l_{50}$ = + 1% | 2,209 | 4,168 | 6,664 | 3,616 | 6,267 | 9,529 | 9,720 | 15,360 | 21,920 | 2,052 | 2,768 | 3,641 | 2,420 | 3,615 | 5,098 | 3,995 | 5,963 | 8,189 |
| $l_{50}$ = - 1% | 2,184 | 4,122 | 6,591 | 3,578 | 6,201 | 9,429 | 9,622 | 15,210 | 21,700 | 2,028 | 2,736 | 3,599 | 2,393 | 3,576 | 5,042 | 3,953 | 5,902 | 8,106 |
| *r* = +1% | 2,153 | 4,064 | 4,604 | 3,533 | 6,124 | 9,312 | 9,525 | 15,050 | 21,480 | 1,999 | 2,697 | 3,548 | 2,363 | 3,531 | 4,980 | 3,915 | 5,845 | 8,028 |
| *r* = -1% | 2,240 | 4,228 | 6,758 | 3,663 | 6,347 | 9,649 | 9,820 | 15,520 | 22,140 | 2,082 | 2,808 | 3,693 | 2,450 | 3,661 | 5,161 | 4,034 | 6,021 | 8,268 |

Source: Authors' own calculation.

**Table 4A** – Sensitivity of the estimated number of centenarians by sex through the Wilmoth's method assuming a non-stable population, Brazil.

| | Female | | | | | | | | | Male | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1991 | | | 2000 | | | 2010 | | | 1991 | | | 2000 | | | 2010 | | |
| | Percentile | | | Percentile | | | Percentile | | | Percentile | | | Percentile | | | Percentile | | |
| | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 | 25 | 50 | 75 |
| Base Model | 1,468 | 2,780 | 4,459 | 2,875 | 5,086 | 7,857 | 8,527 | 14,540 | 22,020 | 1,321 | 1,784 | 2,350 | 1,421 | 2,136 | 3,028 | 3,125 | 4,805 | 6,834 |
| $e_{50}$ = + 1 year | 1,873 | 3,337 | 5,110 | 3,642 | 6,096 | 9,025 | 10,870 | 17,620 | 25,640 | 1,693 | 2,222 | 2,850 | 1,810 | 2,620 | 3,593 | 3,969 | 5,856 | 7,970 |
| $e_{50}$ = - 1 year | 1,153 | 2,318 | 3,895 | 2,274 | 4,245 | 6,838 | 6,701 | 12,000 | 18,920 | 1,032 | 1,434 | 1,939 | 1,116 | 1,743 | 2,553 | 2,464 | 3,947 | 5,734 |
| $_5m_{50}$ = +10% | 1,204 | 2,542 | 4,354 | 2,436 | 4,722 | 7,716 | 7,397 | 13,650 | 21,760 | 996 | 1,434 | 1,995 | 1,113 | 1,813 | 2,731 | 2,549 | 4,239 | 6,291 |
| $_5m_{50}$ = -10% | 1,765 | 3,029 | 4,568 | 3,361 | 5,464 | 7,994 | 9,758 | 15,450 | 22,280 | 1,705 | 2,176 | 2,731 | 1,774 | 2,487 | 3,338 | 3,771 | 5,409 | 7,239 |
| $c$ = +10% | 1,430 | 2,709 | 4,345 | 2,814 | 4,976 | 7,687 | 8,375 | 14,280 | 21,620 | 1,256 | 1,697 | 2,234 | 1,364 | 2,052 | 2,911 | 3,026 | 4,652 | 6,542 |
| $c$ = -10% | 1,508 | 2,853 | 4,579 | 2,939 | 5,198 | 8,026 | 8,682 | 14,800 | 22,430 | 1,384 | 1,979 | 2,471 | 1,479 | 2,223 | 3,150 | 3,228 | 4,965 | 6,982 |
| $v$ = 0.7 | 1,489 | 2,802 | 4,480 | 2,912 | 5,124 | 7,893 | 8,631 | 14,650 | 22,130 | 1,342 | 1,807 | 2,373 | 1,442 | 2,158 | 3,051 | 3,167 | 4,850 | 6,803 |
| $v$ = 0.5 | 1,448 | 2,758 | 4,439 | 2,839 | 5,047 | 7,821 | 8,423 | 16,050 | 21,910 | 1,300 | 1,762 | 2,327 | 1,400 | 2,113 | 3,005 | 3,083 | 4,760 | 6,714 |
| $l_{50}$ = + 1% | 1,477 | 2,796 | 4,485 | 2,891 | 5,114 | 7,900 | 8,573 | 14,610 | 22,140 | 1,329 | 1,795 | 2,364 | 1,429 | 2,148 | 3,046 | 3,142 | 4,832 | 6,795 |
| $l_{50}$ = - 1% | 1,460 | 2,764 | 4,434 | 2,859 | 5,057 | 7,813 | 8,480 | 14,460 | 21,900 | 1,313 | 1,774 | 2,336 | 1,413 | 2,123 | 3,011 | 3,108 | 4,778 | 6,721 |
| $r_1$ and $r_3$ = +1% | 1,427 | 2,702 | 4,335 | 2,792 | 4,939 | 7,631 | 8,252 | 14,070 | 21,310 | 1,238 | 1,733 | 2,282 | 1,380 | 2,074 | 2,942 | 3,029 | 4,657 | 6,551 |
| $r_1$ and $r_3$ = -1% | 1,511 | 2,860 | 4,587 | 2,961 | 5,237 | 8,089 | 8,811 | 15,020 | 22,750 | 1,361 | 1,837 | 2,419 | 1,463 | 2,199 | 3,118 | 3,224 | 4,957 | 6,972 |
| $r_2$ = +1% | 1,475 | 2,793 | 4,479 | 2,887 | 5,107 | 7,889 | 8,557 | 14,590 | 22,100 | 1,327 | 1,792 | 2,360 | 1,427 | 2,145 | 3,041 | 3,136 | 4,822 | 6,782 |
| $r_2$ = -1% | 1,462 | 2,768 | 4,440 | 2,863 | 5,065 | 7,825 | 8,497 | 14,490 | 21,940 | 1,315 | 1,776 | 2,339 | 1,415 | 2,127 | 3,016 | 3,114 | 4,788 | 6,735 |

Source: Authors' own calculation.